Unification of MLSE Receivers and Extension to Time-Varying Channels

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***Abstract*— Forney and Ungerboeck have each developed maximum-likelihood sequence estimation (MLSE) receivers for intersymbol interference (ISI) channels. The Forney receiver uses a whitened matched filter, followed by a sequence estimation algorithm using the Euclidean distance metric. The Ungerboeck receiver uses a matched filter, followed by a sequence estimation algorithm using a modified metric. In this paper a unified development of both receivers is given, in which each receiver is derived from the other. By deriving the Ungerboeck receiver from the Forney receiver, we show that the whitening operation is canceled in the Euclidean distance metric, leaving the modified metric. In addition, the Ungerboeck receiver is extended to the case of a time-varying known channel. When the channel is unknown, decision-directed channel estimation is assumed, which requires channel prediction to account for the decision delay. It is shown that the Ungerboeck receiver requires additional channel prediction, degrading performance due to prediction uncertainty. To solve this problem, two alternative receiver forms are developed which do not require additional prediction, though computational complexity is increased. Performance and complexity of the receiver forms are compared for the IS-136 digital cellular time-division multiple-access (TDMA) standard.**

***Index Terms*— Adaptive equalizers, equalizers, fading channels, maximum likelihood estimation, radio receivers, sequence estimation, time-varying channels.**

I. Introduction

DIGITAL cellular and personal communication systems (PCS's) based on digital advanced mobile phone service (D-AMPS, IS-136) and Global System for Mobile communication (GSM) require an equalizer to handle intersymbol interference (ISI) arising from time dispersion. Typically, nonlinear equalization such as maximum-likelihood sequence estimation (MLSE) [1]–[3] is used in such channels, and MLSE approaches have been studied in particular for the D-AMPS channel [4]–[7]. Most MLSE receivers used in ISI channels are based on one of two classic approaches provided by Forney [1] and Ungerboeck [2]. The purpose of this paper is to relate these two classic MLSE approaches and to extend them to time-varying channels.

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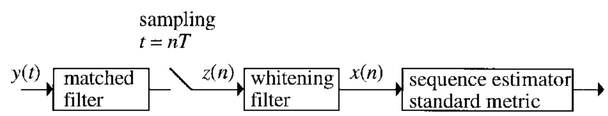

Fig. 1. Forney receiver.

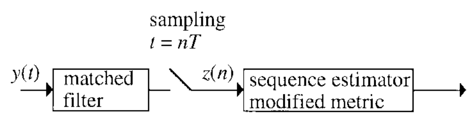

Fig. 2. Ungerboeck receiver.

The Forney receiver [1] was developed by first filtering and sampling the received signal to produce sufficient statistics, then applying MLSE to detect the transmitted symbol sequence. Fig. 1 illustrates the baseband portion of the receiver, which consists of a matched filter, a sampling operation, a whitening filter, and a sequence estimation algorithm, such as the Viterbi algorithm [3]. In [1] the two filtering operations are combined to form a whitened matched filter, and the sequence estimation algorithm employs the standard Euclidean distance metric.

The Ungerboeck receiver [2] was developed from a continuous time representation of the received signal, and a block diagram is shown in Fig. 2. The baseband portion of the receiver comprises a matched filter, a sampling operation, and a sequence estimation algorithm which employs a modified metric.

The first part of this paper unifies the Forney and Ungerboeck receivers through a step-by-step development of the two forms. First, the Forney and Ungerboeck receivers are derived from the likelihood function in [2], showing their mathematical equivalence. Next, the Ungerboeck receiver is derived from the Forney receiver. This approach provides an interesting insight in that the Euclidean distance metric can be rewritten to include a filtering operation that cancels the whitening operation required in the Forney receiver.

Most practical channels are unknown and time-varying, such as channels in mobile radio applications. The MLSE solution for unknown channel coefficients which have known distribution (Gaussian) and known autocorrelation can be found in [9] and [10]. However, the complexity of this solution can be high because it depends not only on the length of the channel impulse response but also on the time correlation of each channel coefficient. A practical suboptimal solution is to employ MLSE receivers based on known channel coefficients in conjunction with adaptive channel estimation [11]. In this case an estimate of the channel is updated using tentative decisions on previously transmitted data symbols. Per-survivor processing (PSP) techniques can be used to minimize estimation delay (see, for example, [12], [13], and the list of early references provided in [14]). Linear and decision feedback equalization, using the zero-forcing criterion, have already been extended to known time-varying channels [15].

A second purpose of this paper is to develop MLSE receivers for time-varying channels, to be used in conjunction with adaptive channel estimation. First, the Ungerboeck receiver is extended to the case of known time-varying channels. When the channel is unknown, channel estimation using decision feedback is assumed. One case of interest is the mobile communications channel, in which the channel comprises a time-invariant transmit filter and a time-varying dispersive propagation medium. For this channel, it is shown that the Ungerboeck receiver requires a set of channel estimates corresponding to future symbol periods. With imperfect channel predictions, as is the case in a noisy fading channel, the performance degrades.

To solve this problem, two alternative receiver forms are derived which minimize the need for channel prediction. The first form, referred to as the "direct form," comprises a time-invariant receive filter matched to the transmit filter, a sampler, and MLSE that operates on the sampled data. This form is more complex than the Ungerboeck form but it minimizes channel prediction. While this form is commonly used [12], [13], the formal derivation indicates under what assumptions the form is optimal. A second novel form, referred to as the "partial Ungerboeck form," is derived from the direct form. It is less complex than the direct form but it has equivalent performance. It is demonstrated via simulation that the standard direct form and the partial Ungerboeck form provide superior performance to the standard Ungerboeck form when channel estimation is included.

The paper is organized as follows. Ungerboeck's receiver formulation is reviewed in Section II with added detail. In Sections III and IV Forney's receiver is derived from Ungerboeck's formulation and vice versa. In Section V the Ungerboeck receiver is extended to the time-varying channel case. In Section VI the specific case of a channel comprising a time-invariant transmit filter and a time-varying transmission medium is considered, and two MLSE receiver forms that minimize channel prediction are derived. Performance and complexity of the receiver forms are compared for the IS-136 digital cellular time-division multiple-access (TDMA) standard in Section VII. Section VIII concludes this paper.

II. Ungerboeck's Receiver Formulation

The system model in [2] consists of a transmitter, transmission medium, and receiver. The receiver converts the radio signal to a complex-valued baseband signal. It is assumed that the complex information symbol sequence $\{a_n\}$ passes through a linear time-invariant channel with impulse response $h(\tau)$, and is received as

$$y(t) = \sum_n a_n h(t - nT) + w(t) \tag{1}$$

where $w(t)$ is a stationary white complex Gaussian noise process (colored noise is considered in [2] as well). The received signal $y(t)$ is collected over a finite time interval, denoted I.

The MLSE receiver finds the hypothetical set of information symbols $\{\alpha_n\}$ that maximizes the likelihood of the received data, given that $\{\alpha_n\}$ was transmitted. This is equivalent to maximizing the log-likelihood function which, ignoring constant scaling factors and additive terms, reduces to the form [2]

$$\begin{aligned} J_H &= -\int_{t\in I} |y(t) - y_H(t)|^2\, dt \\ &= -\int_{t\in I} \left| y(t) - \sum_n \alpha_n h(t-nT) \right|^2 dt \end{aligned} \tag{2}$$

where H is the hypothesis corresponding to the information symbol set $\{\alpha_n\}$. As in [2], it is assumed that $y(t)$ has been bandlimited in the receiver front end, using a bandwidth much larger than the signal bandwidth, so that the integral in (2) is well defined. The expression in (2) is referred to as the sequence estimation metric.

The first step is to *expand the log-likelihood function (step 1)*, giving

$$\begin{aligned} J_H &= -\int_{t\in I} \left[y(t) - \sum_n \alpha_n h(t-nT) \right]^* \\ &\quad \cdot \left[y(t) - \sum_k \alpha_k h(t-kT) \right] dt \\ &= A + B_H + C_H \end{aligned} \tag{3}$$

where

$$A = -\int_{t\in I} |y(t)|^2\, dt \tag{4}$$

$$B_H = \int_{t\in I} 2\mathrm{Re}\left\{ \sum_n \alpha_n^* h^*(t-nT) y(t) \right\} dt \tag{5}$$

$$C_H = -\int_{t\in I} \sum_n \sum_k \alpha_n^* \alpha_k h^*(t-nT) h(t-kT)\, dt. \tag{6}$$

Term A is independent of the sequence hypothesis H and can be omitted.

The next step is to *interchange integration with summation (step 2)* in (5) and (6), giving

$$B_H = 2\mathrm{Re}\left\{ \sum_n \alpha_n^* z(n) \right\} \tag{7}$$

$$\begin{aligned} C_H &= -\sum_n \sum_k \alpha_n^* \alpha_k r_{hh}(n-k) \\ &= -\sum_n \sum_k \alpha_n^* \alpha_k s(n-k) \end{aligned} \tag{8}$$

where

$$z(n) = \int_{t\in I} h^*(t-nT) y(t)\, dt \tag{9}$$

$$s(n-k) = r_{hh}(n-k) = \int_{t+nT\in I} h^*(t) h[t+(n-k)T]\, dt \tag{10}$$

which are referred to as the matched filter outputs and the "*s-parameters*," respectively.

Interpreting (8) as a matrix summation, the double sum can be split into a single summation along the matrix diagonal and a double summation, in which partial rows and partial columns are combined. Mathematically, this can be expressed as

$$\sum_r \sum_c m_{rc} = \sum_r m_{rr} + \sum_r \sum_{\substack{c \\ c<r}} m_{rc} + m_{cr} \tag{11}$$

where the indexes r and c refer to row and column indexes of the matrix.

The third step is to *rearrange the double summation (step 3)* in (8) by applying (11) and using the fact that $s(k-n) = s^*(n-k)$, which gives

$$C_H = -\sum_n \alpha_n^* \alpha_n s(0) - \sum_n \sum_{\substack{k \\ k<n}} 2\mathrm{Re}\{\alpha_n^* \alpha_k s(n-k)\}. \tag{12}$$

Finally, the summation over k can be replaced by a summation over $\ell = n - k$, giving

$$C_H = -\sum_n \alpha_n^* \alpha_n s(0) - \sum_n \sum_{\substack{\ell \\ \ell>0}} 2\mathrm{Re}\{\alpha_n^* \alpha_{n-\ell} s(\ell)\}. \tag{13}$$

Collecting the results of (7) and (13), the sequence estimation metric can be expressed as

$$J_H = \sum_n M_H(n) \tag{14}$$

where the branch metric is defined as

$$M_H(n) = \mathrm{Re}\left\{ \alpha_n^* \left[2z(n) - s(0)\alpha_n - 2 \sum_{\substack{\ell \\ \ell>0}} \alpha_{n-\ell} s(\ell) \right] \right\}. \tag{15}$$

The hypothesis H which maximizes (14) can be determined using a sequence estimation algorithm such as the Viterbi algorithm, in which each iteration of the algorithm uses $M_H(n)$.

III. Forney's Receiver from Ungerboeck's Formulation

In this section Forney's receiver for ISI channels [1] is derived from the same continuous-time log-likelihood function used to derive the Ungerboeck receiver. This formulation of Forney's receiver considers complex channel coefficients, which is a simple extension of the real coefficient case described in [1].

In the previous section the sequence estimation metric in (2) was shown to comprise two terms: B_H in (7) and C_H in (8), which depend on $z(n)$ in (9) and $s(n-k)$ in (10). A change in notation from $s(n-k)$ to $r_{hh}(n-k)$ is used to provide consistency with [1] and to stress that $r_{hh}(n-k)$ is the channel deterministic autocorrelation. Substituting (1) in (9) gives the model

$$z(n) = \sum_i a_i r_{hh}(n-i) + u(n) \tag{16}$$

where

$$u(n) = \int_{t \in I} h^*(t - nT) w(t)\, dt. \tag{17}$$

Next, consider the spectral factorization of $r_{hh}(m)$. For consistency with [1], the D-transform is used instead of the more prevalent z-transform, though the two are easily related by $D = z^{-1}$. Spectral factorization gives

$$R_{hh}(D) = F(D) F^*(D^{-1}) \tag{18}$$

and the inverse D-transform gives the discrete convolution

$$r_{hh}(m) = \sum_d f(m-d) f^*(-d) = \sum_p f(p) f^*(p-m). \tag{19}$$

The noise sequence $u(n)$ has a statistical autocorrelation function of the form

$$r_{uu}(m) = E\{u(n+m) u^*(n)\} = K r_{hh}(m) \tag{20}$$

where the constant K depends on the noise power spectral density and the gain of the channel. From (18)–(20), $u(n)$ can be generated by passing a white noise sequence $v(n)$ through a filter whose D-transformed impulse response is $F^*(D^{-1})$. Mathematically

$$u(n) = \sum_d f^*(-d) v(n-d). \tag{21}$$

Substituting (19) and (21) into (16) gives the model

$$z(n) = \sum_p f^*(p-n) \left[\sum_i a_i f(p-i) + v(p) \right]. \tag{22}$$

Substituting (22) into (7) gives the following alternative expression for B_H:

$$B_H = 2\mathrm{Re}\left\{ \sum_p x_H^*(p) x(p) \right\} \tag{23}$$

where

$$x(p) = \sum_i a_i f(p-i) + v(p) \tag{24}$$

$$x_H(p) = \sum_n \alpha_n f(p-n). \tag{25}$$

An alternative expression for C_H is obtained by substituting (19) in (8), giving

$$C_H = -\sum_p |x_H(p)|^2. \tag{26}$$

Adding a constant term, independent of the sequence hypothesis, does not affect performance. Hence, adding a summation over p of $|x(p)|^2$ with the terms in (23) and (26), the sequence estimation metric can be expressed as

$$J_H = -\sum_p |x(p) - x_H(p)|^2 \tag{27}$$

which indicates that MLSE can equivalently be performed using the Euclidean distance metric and discrete-time signal $x(p)$, whose signal model is given in (24).

It is shown in [1] that the discrete-time signal $x(p)$ can be obtained by passing the continuous-time signal $y(t)$ through matched filter, sampling, and whitening operations. The matched filter output is given by (16), and its D-transform, using (21) and (24), is given by

$$Z(D) = \sum_i a_i F(D) D^i F^*(D^{-1}) + V(D) F^*(D^{-1}). \quad (28)$$

The whitening filter has a response given by $1/F^*(D^{-1})$. Applying this filter to (28) gives

$$X(D) = \frac{Z(D)}{F^*(D^{-1})} = \sum_i a_i F(D) D^i + V(D) \quad (29)$$

which is the D-transform of $x(p)$, as defined in (24).

Thus, the MLSE metric can be realized by passing the received signal $y(t)$ through a filter matched to the channel response $h(\tau)$, producing samples $z(p)$. These samples are whitened, giving the sequence $x(p)$, which are processed by a sequence estimation algorithm using the Euclidean distance metric. This series of steps is the Forney receiver [1], illustrated in Fig. 1. Thus, both the Ungerboeck and Forney receivers have been derived from a common formulation.

Observe that the Forney receiver was derived from an intermediate form of the Ungerboeck sequence estimation metric by partitioning this metric into a whitening operation followed by the Euclidean distance metric. One can interpret the whitening operation as a consequence of partitioning the metric to include the Euclidean distance metric. Using other partitioning approaches, it is possible to develop other filtering operations and metric expressions, such as those proposed in [8]. The partition that leads to the Forney receiver is of particular interest, as it impacts performance when channel estimation is used in conjunction with time-varying channels, as will be discussed in Section VI.

IV. Ungerboeck's Receiver from Forney's Formulation

As with (2), the metric in (27) can be modified by applying the three steps employed in Section II. However, now the input is discrete-time signal $x(p)$, and the channel model is given in (24). The first step is to substitute (24) in (27) and expand. Dropping the first term leaves

$$B'_H = \sum_p 2\mathrm{Re}\left\{ \sum_n \alpha_n^* f^*(p-n) x(p) \right\} \quad (30)$$

$$C'_H = -\sum_p \sum_n \sum_k \alpha_n^* \alpha_k f^*(p-n) f(p-k). \quad (31)$$

The second step, rearranging the summations in (30) and (31), gives, using (10) and (19)

$$B'_H = 2\mathrm{Re}\left\{ \sum_n \alpha_n^* z'(n) \right\} \quad (32)$$

$$\begin{aligned} C'_H &= -\sum_n \sum_k \alpha_n^* \alpha_k \sum_p f^*(p-n) f(p-k) \\ &= -\sum_n \sum_k \alpha_n^* \alpha_k s(n-k) \end{aligned} \quad (33)$$

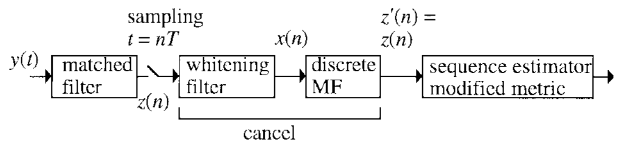

Fig. 3. Modified Forney receiver.

where

$$z'(n) = \sum_p f^*(p-n) x(p). \quad (34)$$

The third step is to rearrange the summation in (33) using (11) and the fact that $s(k-n) = s^*(n-k)$, which gives

$$C'_H = -\sum_n \alpha_n^* \alpha_n s(0) - \sum_n \sum_{\substack{\ell \\ \ell > 0}} 2\mathrm{Re}\{\alpha_n^* \alpha_{n-\ell} s(\ell)\}. \quad (35)$$

Collecting the results of (32) and (35), the sequence estimation metric is given by (14), where

$$M_H(n) = \mathrm{Re}\left\{ \alpha_n^* \left[2z'(n) - s(0)\alpha_n - 2 \sum_{\substack{\ell \\ \ell > 0}} \alpha_{n-l} s(\ell) \right] \right\}. \quad (36)$$

This modified form of the Forney receiver is shown in Fig. 3.

To complete the derivation of the Ungerboeck receiver, it remains to show that $z'(n)$ is equal to $z(n)$, which is achieved by taking the D-transform of (34) and substituting (24), giving

$$Z'(D) = F^*(D^{-1}) X(D) = F^*(D^{-1}) \frac{Z(D)}{F^*(D^{-1})} = Z(D). \quad (37)$$

This alternative derivation of the Ungerboeck receiver provides an interesting insight. As shown in (37) and Fig. 3, the discrete-time filtering operation of (34) cancels the whitening operation used to form $x(p)$ from $z(p)$. This is why the Ungerboeck form does not require a whitening operation. One can interpret the Ungerboeck receiver as the result of partitioning Forney's Euclidean distance metric into a filtering operation and a modified metric. The filtering operation undoes the whitening operation, leaving a matched filter and a modified metric.

V. Extension to Time-Varying Channels

In this section the Ungerboeck receiver is extended to the case of a time-varying channel [16]. For such a channel, the output $y(t)$ can be related to the input, in this case $\sum_n a_n \delta(t - nT)$, by means of a time-varying impulse response $h(\tau; t)$ in which τ denotes delay and t denotes time variation. Additive noise $w(t)$ is present at the channel output, so that the received signal is given by

$$\begin{aligned} y(t) &= \int h(\tau; t) \left[\sum_n a_n \delta(t - \tau - nT) \right] d\tau + w(t) \\ &= \sum_n a_n h(t - nT; t) + w(t). \end{aligned} \quad (38)$$

Note that $h(\tau; t)$ models both the transmit pulse shaping and the transmission medium response. The sequence estimation metric is similar to (3), giving

$$J_H = -\int_{t \in I} |y(t) - y_H(t)|^2 \, dt = -\int_{t \in I} \left| y(t) - \sum_n \alpha_n h(t - nT; t) \right|^2 dt. \quad (39)$$

Once again, the three steps used to derive the Ungerboeck receiver are applied. Applying the first step gives a metric that consists of two terms B_H and C_H given by

$$B_H = 2\mathrm{Re}\left\{ \sum_n \alpha_n^* z(n) \right\} \quad (40)$$

$$C_H = -\sum_n \sum_k \alpha_n^* \alpha_k s(n-k, n) \quad (41)$$

where

$$z(n) = \int_{t \in I} h^*(t - nT; t) y(t) \, dt \quad (42)$$

$$s(\ell, n) = \int_{t+nT \in I} h^*(t; t+nT) h(t + \ell T; t + nT) \, dt. \quad (43)$$

Applying the remaining two steps in deriving the Ungerboeck receiver (see Section II) gives the branch metric for a known time-varying channel

$$M_H(n) = \mathrm{Re}\left\{ \alpha_n^* \left[2z(n) - s(0, n)\alpha_n - 2 \sum_{\substack{\ell \\ \ell > 0}} \alpha_{n-\ell} s(\ell, n) \right] \right\}. \quad (44)$$

Observe that the matched filter response and *s-parameters* vary with time.

VI. Practical Receivers for Mobile Communications

In mobile communications it is common to consider the channel as comprising a time-invariant transmit filter (pulse-shaping filter), with impulse response $q(\tau)$, followed by a time-varying transmission medium with impulse response $g(\tau; t)$, as illustrated in Fig. 4. If the transmitted signal is finite in bandwidth and employs no excess bandwidth, then an arbitrary medium response can be modeled by an equivalent symbol-spaced delay response given by [17]

$$g_{\mathrm{equiv}}(\tau; t) = \sum_j c(jT; t) \delta(\tau - jT) \quad (45)$$

where

$$c(j; t) = \int g(\tau; t) \frac{\sin[\pi(jT - \tau)/T]}{\pi(jT - \tau)/T} \, d\tau. \quad (46)$$

Symbol-spaced channel models are commonly used to develop MLSE receivers [12], [13], and such models yield reasonable performance when signal excess bandwidth is small. However, symbol-spaced equalization of excess bandwidth signals is suboptimal, as fractionally spaced equalization is optimal. With the corresponding assumptions on signal bandwidth and pulse shaping, it is straightforward to extend the results of this section to fractionally spaced equalization.

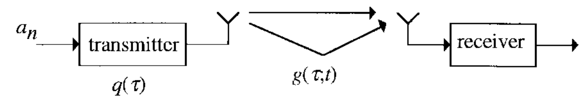

Fig. 4. Mobile communications model.

In general, $g(\tau, t)$ is continuous in τ, so that there is an infinite number of nonzero symbol-spaced channel taps. It is assumed that $g(\tau, t)$ can be well approximated by J nonzero channel coefficients, so that the overall channel response (transmit filter + medium) becomes

$$h(\tau; t) = \sum_{j=0}^{J-1} c(j; t) q(\tau - jT). \quad (47)$$

A. Ungerboeck Receiver

Substituting (47) into (42) and (43) gives

$$z(n) = \sum_{j=0}^{J-1} \int_{t \in I} c^*(j; t) q^*[t - (j+n)T] y(t) \, dt \quad (48)$$

$$s(\ell, n) = \sum_{j=0}^{J-1} \sum_{k=0}^{J-1} \int_{t+nT \in I} c^*(j; t+nT) c(k; t+nT) \cdot q^*(t - jT) q[t + (\ell - k)T] \, dt. \quad (49)$$

It is assumed that the time variation of each channel coefficient $c(j; t)$ is slow relative to the duration of the transmit pulse shape $q(\tau)$. This assumption allows for standard radio receiver design practices but it introduces suboptimality commensurate with the rate of channel variation relative to the symbol rate. With this assumption, (48) and (49) become

$$z(n) \approx \sum_{j=0}^{J-1} c^*(j; n+j) Y(n+j) \quad (50)$$

$$s(\ell, n) \approx \sum_{j=0}^{J-1} \sum_{k=0}^{J-1} c^*(j; n+j) c(k; n+k-\ell) \cdot r_{qq}(\ell + j - k) \quad (51)$$

where

$$Y(n) = \int_{t \in I} q^*(t - nT) y(t) \, dt \quad (52)$$

is a sequence of symbol-spaced data samples at the output of a fixed front-end receive filter, matched to the transmit filter, and $r_{qq}(m)$ is the deterministic autocorrelation function for pulse shape $q(t)$. Observe that both the pulse shape autocorrelation and the medium impulse response impact the sequence estimation metric.

Assuming the transmit pulse shape convolved with itself is Nyquist $[r_{qq}(m) = \delta(m)]$, then

$$s(\ell, n) \approx \sum_{j=0}^{J-1-\ell} c^*(j; n+j) c(j+\ell; n+j) \quad (53)$$

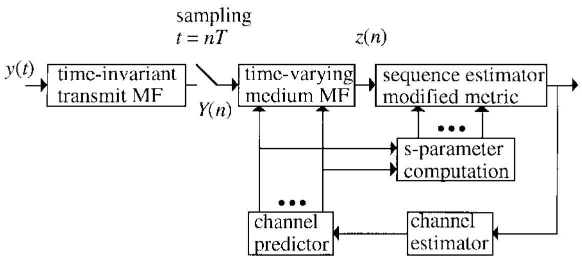

Fig. 5. Adaptive Ungerboeck receiver.

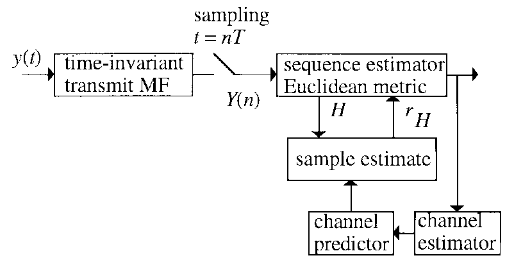

Fig. 6. Adaptive direct form receiver.

and the model for the received samples becomes

$$Y(n) = \sum_{j=0}^{J-1} c(j;\, n)a_{n-j} + W(n) \tag{54}$$

where $W(n)$ is a sequence of independent Gaussian noise samples.

Observe that the continuous-time received signal $y(t)$ can be first passed through a receive filter matched to the transmit filter, giving rise to a discrete-time signal $Y(n)$. Assuming symbol-spaced equalization and Nyquist pulse shaping, the Ungerboeck receiver forms metrics using $Y(n)$ and knowledge of the time-varying medium coefficients $c(j;\, n)$. For the assumptions given, the medium coefficients are equivalent to the composite response comprising the transmit filter, the medium, and the receive filter.

At iteration n the Ungerboeck receiver computes $M_H(n)$ for various symbol hypotheses. From (50) and (53), this requires knowledge of the time-varying channel coefficients at times nT through $(n + J - 1)T$. This knowledge may be difficult to obtain when decision feedback is used to estimate the channel [11]. Typically, after iteration n is completed, tentative decisions on symbols $s(n - \Delta)$ are made, where Δ is an update decision delay. Using these tentative decisions, the channel estimates at time $n - \Delta$ are updated, giving values corresponding to time $n - \Delta + 1$. The channel coefficients should then be predicted using some prediction technique to time $n + 1$, the next iteration of the equalizer [18]. With the Ungerboeck form, predictions would be needed at times $n+1$ through $n + J$. The baseband processor of the receiver is illustrated in Fig. 5.

With typical channel estimation and prediction algorithms, the accuracy of the prediction decreases with the number of steps over which the prediction is made. Thus, from a practical implementation point of view, there is interest in a receiver form which minimizes the prediction of channel coefficients.

B. Direct Form

A form which minimizes prediction, referred to as the "direct form," can be derived from the Ungerboeck form. Substituting (50) and (53) into (40) and (41), respectively, gives

$$B_H = 2\text{Re}\left\{\sum_n Y_H^*(n)Y(n)\right\} \tag{55}$$

$$C_H = -\sum_n |Y_H(n)|^2 \tag{56}$$

where

$$Y_H(n) = \sum_{j=0}^{J-1} c(j;\, n)\alpha_{n-j}. \tag{57}$$

From (3), with $A = -\sum_n |Y(n)|^2$, the branch metric becomes

$$M_H(n) = -|Y(n) - Y_H(n)|^2. \tag{58}$$

The final metric is given by (14) with $M_H(n)$ defined by (58).

The direct form processes samples $Y(n)$ sequentially. At iteration n, branch metric $M_H(n)$ depends only on channel coefficients at time n, as seen in (57) and (58), minimizing the amount of channel prediction. The direct form is well known, and the above derivation provides a formal development of the receiver. Note that the direct form is optimal when there is zero excess bandwidth and Nyquist pulse shaping. The direct form is illustrated in Fig. 6.

Under the assumptions given, the direct form can be viewed as a Forney receiver (see acknowledgment). The matched filter comprises two filters: one matched to the pulse shape (52) and one matched to the time-varying channel (50). The former gives symbol-spaced white noise samples, whereas the latter colors these noise samples. The whitening filter undoes the latter, so that the whitened matched filter is simply the filter matched to the pulse shape.

C. Partial Ungerboeck Form

An alternative form is obtained from the direct form by applying two of the three steps used to derive the Ungerboeck form. The first step is performed by substituting (57) and (58) into (14), expanding the metric, and dropping the hypothesis independent term, leaving the terms defined by (55) and (56). Substituting (57) in (55) and (56) gives

$$\begin{aligned} B_H &= \sum_n 2\text{Re}\left\{\sum_{j=0}^{J-1} \alpha_{n-j}^* c^*(n,\, j)Y(n)\right\} \\ &= \sum_n 2\text{Re}\left\{\sum_{j=0}^{J-1} \alpha_{n-j}^* Z(n,\, j)\right\} \end{aligned} \tag{59}$$

$$C_H = -\sum_n \sum_{j=0}^{J-1}\sum_{k=0}^{J-1} c^*(j;\, n)c(k;\, n)\alpha_{n-j}^*\alpha_{n-k} \tag{60}$$

where $Z(n,\, j)$ are referred to as "*Z-parameters*."

The second step is not performed. Instead, a modified form of the third step is applied, in which the expression in (11) is

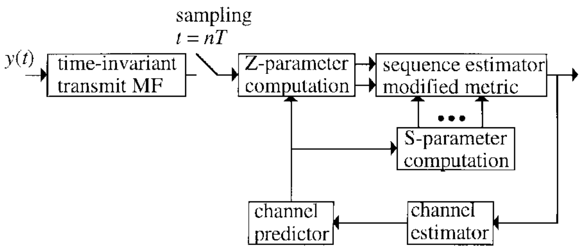

Fig. 7. Adaptive partial Ungerboeck receiver.

used with $c < r$ being replaced by $c > r$. As a result, (60) becomes

$$C_H = -\sum_n \sum_{j=0}^{J-1} \mathrm{Re}\Bigg\{\alpha_{n-j}^* \cdot \left[S(0, n, j)\alpha_{n-j} + 2\sum_{\ell=1}^{J-1-j} S(\ell, n, j)\alpha_{n-j-\ell}\right]\Bigg\} \tag{61}$$

where

$$S(\ell, n, j) = c^*(j; n)c(j+\ell; n) \tag{62}$$

which are referred to as "*S-parameters*." Putting these results together, the accumulated metric is given by (14), and the branch metric is given by

$$M_H(n) = \sum_{j=0}^{J-1} \mathrm{Re}\Bigg\{\alpha_{n-j}^* \Bigg[2Z(n, j) - S(0, n, j)\alpha_{n-j} - 2\sum_{\ell=1}^{J-1-j} \alpha_{n-j-\ell}S(\ell, n, j)\Bigg]\Bigg\}. \tag{63}$$

This form is similar to the Ungerboeck form, except that the summation over j appears on the outside of the branch metric rather than the inside. Because only some of the steps used in deriving the Ungerboeck receiver are employed, this form is referred to as the "partial Ungerboeck form." The partial Ungerboeck form is illustrated in Fig. 7.

VII. Comparison of Receiver Forms

The three receiver forms developed in the previous section are compared in terms of performance and complexity for the IS-136 digital cellular TDMA standard. It is shown that while the standard Ungerboeck form minimizes complexity, it suffers a performance degradation when practical channel estimation is used. The standard direct form and the partial Ungerboeck form have higher complexity, but perform better with practical channel estimation.

A. The IS-136 Example

The IS-136 digital cellular standard [19] is used to illustrate the performance and complexity tradeoffs of the proposed receivers. In the IS-136 standard, each 30-kHz downlink (base station to mobile station) radio channel is divided into six time slots, each slot having the format shown in Fig. 8. The

bits	28	12	130	12	130	1	11
field	SYNC	SACCH	DATA	CDVCC	DATA	RSVD	CDL

Fig. 8. IS-136 traffic channel slot structure.

slot begins with a known synchronization pattern, followed by various data fields. A full-rate traffic channel uses two slots.

The modulation is π/4–shift differential quadrature phase-shift keying (π/4-DQPSK), and the transmit pulse shaping is root-raised-cosine with 35% rolloff. The symbol rate is 24.3 kbaud and each slot is 162 symbols long (6.67 ms). A 900-MHz carrier and a vehicle speed of 100 km/h are assumed, so that the medium response varies significantly within a single time slot (83.3-Hz Doppler spread).

For this example, the medium response consists of two independently fading channel taps, separated by one symbol period, which is modeled at the receiver with two taps ($J = 2$). There are four states in the sequence estimation process, one for each possible previous symbol value. The path history of each state is truncated, so that symbol decisions are made with a delay of six symbol periods.

Channel estimation is based on the classic least-mean-square (LMS) approach, using an empirically derived step size of 0.155. Channel estimates are initialized using correlations to the synchronization pattern, then refined using LMS over the synchronization field. In adapting the channel estimates over the data fields, two approaches are considered. In the first, a single model of the channel is updated after each sequence estimation iteration by determining the best state, and using the symbols in its path history two and three symbol periods prior to the current time instant ($\Delta = 2$). In the second approach, PSP is employed, so that there is a channel model per state. After each sequence estimation iteration, each channel model is updated based on the state and one prior symbol in the path history.

B. Performance

If the medium impulse response is known and time-invariant, then all MLSE receiver forms have the same performance, which can be estimated using the results in [1] and [2]. Additional analysis is needed for the case of known time-varying channels. When adaptive channel estimation is employed, performance can differ between forms and depends on the channel estimation approach employed. The direct and partial Ungerboeck forms have the same performance, so only the direct and Ungerboeck forms are examined.

The direct and Ungerboeck forms were initially simulated using perfect knowledge of the channel, except that only knowledge of the channel at discrete time k was available at iteration k. The results, given by the first two curves in Fig. 9, indicate that there is negligible loss in the Ungerboeck form relative to the other forms by approximating $c(1; k+1)$ with $c(1; k)$. However, when the channel is estimated, the loss can be significant, as illustrated by the remaining simulation curves in Fig. 9. At 3% bit-error rate (BER) the Ungerboeck form loses 0.4 dB relative to the direct form, when both employ the first channel estimation approach. For the second channel

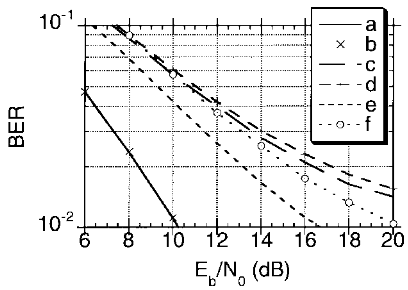

Fig. 9. Performance comparison of MLSE receivers. (a) Direct form, known channel. (b) Ungerboeck form, known channel. (c) Direct form, estimated channel. (d) Ungerboeck form, estimated channel. (e) Direct form, estimated channel with PSP. (f) Ungerboeck form, estimated channel with PSP.

estimation approach, the loss of the Ungerboeck form is 1.7 dB. At lower BER levels the loss is larger. Thus, in this example, the direct and partial Ungerboeck forms provide a significant improvement in performance.

C. Complexity

Relative complexity is defined as the number of arithmetic operations required to compute the metrics $M_H(n)$ for each iteration n. A scalar addition or scalar multiplication constitutes an arithmetic operation. For π/4-DQPSK, all symbols have a constant modulus, so that computation of the terms $s(0, n)\alpha_n$ and $S(0, n, j)\alpha_{n-j}$ in (44) and (63), respectively, can be omitted.

Using these simplifications, the Ungerboeck form requires 100 arithmetic operations, the partial Ungerboeck form 126 arithmetic operations, and the direct form 160 operations. In this example it is interesting to see that minimization of the channel prediction, as with the direct form, increases complexity. The partial Ungerboeck form provides a good tradeoff between minimization of channel prediction and increase in complexity.

In general, complexity calculation is more involved and depends on the hardware implementation and memory requirements. Further, simplifications in the metric and reordering of operations can be done to reduce the number of computations per iteration in all forms. Hence, the conclusions of the above example do not change significantly.

VIII. Conclusion

In this paper we have unified the theory of MLSE for channels with ISI. We showed that the MLSE receivers derived by Forney and Ungerboeck are mathematically equivalent, although their realizations are quite different. We also showed a method by which each receiver could be derived from the other. The Ungerboeck receiver can be derived by further developing the MLSE (nonlinear) part of the Forney receiver, resulting in a second filtering operation that cancels the whitening operation. This is why the Ungerboeck receiver does not require the whitening filter operation used in the Forney receiver. Also, the whitening operation in the Forney receiver can be viewed as a result of partitioning the Ungerboeck metric to include the Euclidean distance metric.

Having developed the basic theory of the two receivers, we have extended their application to time-varying channels such as the mobile radio channel. For this channel, it was shown that the Ungerboeck receiver requires significant channel prediction. Two alternative receiver forms were developed which minimize channel prediction but increase computational complexity. The "partial Ungerboeck" form offers a compromise in complexity and performance.

Acknowledgment

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References

[1] G. D. Forney, "Maximum-likelihood sequence estimation of digital sequences in the presence of intersymbol interference," *IEEE Trans. Inform. Theory*, vol. IT-18, pp. 363–378, May 1972.
[2] G. Ungerboeck, "Adaptive maximum likelihood receiver for carrier modulated data transmission systems," *IEEE Trans. Commun.*, vol. COM-22, pp. 624–635, May 1974.
[3] G. D. Forney, Jr., "The Viterbi algorithm," *Proc. IEEE*, vol. 61, pp. 268–278, Mar. 1973.
[4] G. Larsson, B. Gudmundson, and K. Raith, "Receiver performance of the North American digital cellular system," in *Proc. VTC'91*, St. Louis, MO, May 19–22, 1991, pp. 1–6.
[5] R. D. Koilpillai, S. Chennakeshu, and R. T. Toy, "Low complexity equalizers for U.S. digital cellular system," in *Proc. VTC'92*, Denver, CO, May 10–13, 1992, pp. 744–747.
[6] Q. Liu and Y. Wan, "An adaptive maximum-likelihood sequence estimation receiver with dual diversity combining/selection," in *Int. Symp. Personal, Indoor and Mobile Radio Communications*, Boston, MA, Oct. 19–21, 1992, pp. 245–249.
[7] P. K. Shukla and L. F. Turner, "Examination of an adaptive DFE and MLSE/near-MLSE for fading multipath radio channels," *Proc. Inst. Elect. Eng.*, vol. 139, pt. I, pp. 418–428, Aug. 1992.
[8] L. C. Barbosa, "Maximum likelihood sequence estimators: A geometric view," *IEEE Trans. Inform. Theory*, vol. 35, pp. 419–427, Mar. 1989.
[9] T. Kailath, "Correlation detection of signals perturbed by a random channel," *IEEE Trans. Inform. Theory*, vol. IT-6, pp. 361–366, June 1960.
[10] X. Yu and S. Pasupathy, "Innovations-based MLSE for Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 43, pp. 1534–1544, Feb./Mar./Apr. 1995.
[11] J. G. Proakis, *Digital Communications*, 2nd ed. New York: McGraw-Hill, 1989.
[12] N. Seshadri, "Joint data and channel estimation using blind trellis search techniques," *IEEE Trans. Commun.*, vol. 42, pp. 1000–1011, Feb./Mar./Apr. 1994.
[13] H. Kubo, K. Murakami, and T. Fujino, "An adaptive maximum-likelihood sequence estimator for fast time-varying intersymbol interference channels," *IEEE Trans. Commun.*, vol. 42, pp. 1872–1880, Feb./Mar./Apr. 1994.
[14] K. A. Hamied and G. L. Stüber, "An adaptive truncated MLSE receiver for Japanese personal digital cellular, " *IEEE Trans. Veh. Technol.*, vol. 45, pp. 41–50, Feb. 1996.
[15] D. G. Messerschmitt, "A geometric theory of intersymbol interference part I: Zero-forcing and decision-feedback equalization," *Bell Syst. Tech. J.*, vol. 52, pp. 1483–1519, Nov. 1973.
[16] G. E. Bottomley and S. Chennakeshu, "Adaptive MLSE equalization forms for wireless communications," in *Fifth Virginia Tech. Symp. Wireless Personal Communications*, Blacksburg, VA, May 31–June 2, 1995, pp. 20-1–20-12.
[17] H. L. Van Trees, *Detection, Estimation, and Modulation Theory Part III: Radar–Sonar Signal Processing and Gaussian Signals in Noise.* New York: Wiley, 1968.
[18] E. Dahlman, "New adaptive Viterbi detector for fast-fading mobile-radio channels," *Electron. Lett.*, vol. 26, pp. 1572–1573, Sept. 1990.
[19] *TIA/EIA Interim Standard 800 MHz TDMA Cellular—Radio Interface—Mobile Station—Base Station Compatibility—Traffic Channels and FSK Control Channel*, Telecommunications Industry Association Standard TIA/EIA/IS-136.2, Dec. 1994.

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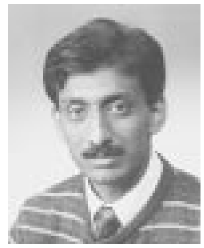

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