A MODIFIED SNR ESTIMATION ALGORITHM BASED ON SINGULAR VALUE DECOMPOSITION

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Abstract

When the traditional SNR estimation algorithm based on singular value decomposition (SVD) is applied to the digital data analysis of spaceflight TT&C system, problem of large deviation arises. This paper analyzes the limitations of SVD, and proposes a modified singular value decomposition (MSVD) SNR estimation algorithm .Before estimating SNR by use of SVD, MSVD does some processing to whiten noise, such as band-pass, down-sampling, and up-sampling. The simulation results show that this algorithm can, within a large range, accurately estimate the actually received SNR, effectively improving the application scope of traditional algorithms.

1 Introduction

SNR is an important characteristic parameter of modulation signal. Accurately estimating the SNR is important for understanding the channel characteristics and performance of spaceflight TT&C equipment. Our predecessors have proposed many effective SNR estimation algorithms, which can be divided into two types -- blind and non-blind estimation, such as MLE (maximum likelihood estimation), SSME (split symbol moments estimate), SNV (squared SNR estimation variance ratio), SVD, etc. Pauluzzi D.R. introduced and compared several algorithms of SNR estimation[1]. Turkboylari estimated signal to interference ratio of time division multiple access (TDMA) system by using the method of signal space projection[2], but it requires a correlation matrix of the received signal created by training sequence. However, these algorithms bring a lot of constraints to users, particularly that the non-blind estimation algorithm greatly limits the scope of application. We must use blind estimations when we haven't obtained enough priori information about the signal in many applications.

SVD is a typical representative of SNR blind-estimation algorithm. Its estimation accuracy was significantly higher than the other algorithms, but the noise power spectral density should be kept flat over the entire frequency domain. However, in the actual TT&C system, digital signal must be processed through a band-pass filter with unideal characteristic before received by IF digital receiver. In addition, the actual sampling frequency is not integer times of band-pass filter bandwidth. For these reasons, noise is no longer flat over the entire frequency domain. Obviously, SVD is improper.

This paper consists of three parts. Firstly, it introduces the traditional SVD algorithm. Secondly, some useful improvements are proposed. At last, the simulation results show that the improved SNR estimation algorithm is effective.

2 SNR blind estimation based on SVD

Supposing under AWGN channel, the receiver receives signal without distortion, then the discrete signal is given by:

$$r(kT_0) = s(kT_0) + w(kT_0) \tag{1}$$

where $s(kT_0)$ represents modulated signal, $w(kT_0)$ represents white gaussian noise, T_0 represents sampling interval. For convenience, the above equation(1) can be simplified as

$$r = s + w \tag{2}$$

Fan Haibo has proposed a blind SNR estimation algorithm based on autocorrelation matrix singular value decomposition[3]. L rank correlation matrix of received signal is given by

$$\begin{aligned} R_{rr} &= E[rr^H] = E[(s+w)(s+w)^H] \\ &= E[ss^H] + E[ww^H] = R_{ss} + R_{ww} \end{aligned} \tag{3}$$

where matrix R_{rr} , R_{ss} ,and R_{ww} are symmetric matrix. Decompose R_{rr} and get the following

$$\begin{aligned} R_{rr} &= R_{ss} + R_{ww} \\ &= U\Lambda_r U^H = U(\Lambda_s + \Lambda_w)U^H \end{aligned} \tag{4}$$

where Λ_s and Λ_w are from signal sequence and noise sequence, the diagonal elements are singular values.

$$\begin{aligned} \Lambda_s &= diag(\lambda_1, \lambda_2, ..., \lambda_d, 0, ..., 0)_{L\times L}, \\ \Lambda_w &= diag(\sigma_w^2, \sigma_w^2, ..., \sigma_w^2)_{L\times L}, \\ \Lambda_r &= diag(\lambda_1 + \sigma_w^2, ..., \lambda_d + \sigma_w^2, \sigma_w^2, ..., \sigma_w^2)_{L\times L} \end{aligned} \tag{5}$$

Assuming that, signal s and noise w are independent to each other. SNR of received signal is given by

$$\rho_{autocor} = 10\lg\frac{\sum_{k=1}^{d}(\lambda_k - \sigma_w^2)}{L\times\sigma_w^2} \qquad (6)$$

where d is signal space dimension, which can be calculated by the minimum description length algorithm (MDL) [4][5]. The steps of this algorithm are as follows:

1. The autocorrelation sequence of received signal is given by

$$r_k = E\{y(n)y(n+k)\} = \frac{1}{N}\sum_{i=1}^{N} y(i)y^*(i+k-1),(k=1,2,3,...,L) \qquad (7)$$

where N represents length of the signal, L represents length of autocorrelation sequence, L is generally set between 40 and 100.

2. Construct the autocorrelation matrix R from the autocorrelation sequence. Decompose R and get the singular values $\{\lambda_1,\lambda_2,\lambda_3,...,\lambda_L\}$.

3. Determine the signal subspace dimension d, define the spherical test function as following:

$$T(m) = \frac{1}{L-m}\cdot\frac{\sum_{i=m+1}^{L}\lambda_i}{(\prod_{i=m+1}^{L}\lambda_i)^{\frac{1}{N-m}}} \qquad (8)$$

MDL equation is given by

$$MDL(m) = K(L-m)\log[T(m)] + 0.5m(2L-m)\log K \qquad (9)$$

d is given by

$$d = \arg\cdot\min_m MDL(m) \qquad (10)$$

4. Estimate average noise power σ_n^2 and signal power p_s

$$\sigma_n^2 = \frac{1}{L-d}\sum_{i=d+1}^{L}\lambda_i \qquad (11)$$

$$p_s = \sum_{i=1}^{d}(\lambda_i - \sigma_n^2) \qquad (12)$$

5. Estimate SNR

$$snr\_est = 10\log_{10}\frac{p_s}{L\cdot\sigma_n^2} \qquad (13)$$

3 Modified blind SNR estimation algorithm

The above algorithm, based on autocorrelation matrix singular value decomposition, can be applied to both base band signals and intermediate frequency signal without prior knowledge. Although its estimation accuracy is significantly higher than other algorithms, the noise power spectral density should be kept flat over the entire frequency domain.

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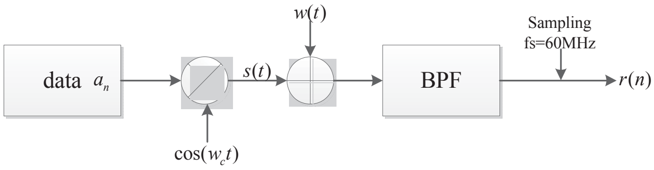

Fig.1: BPSK signal generating model

In order to facilitate the analysis, we have built the BPSK signal generating model as shown in Figure 1. Where $s(t)$ represents modulated signal before shaping filter, B_w represents the bandwidth of band-pass filter.

Simulation parameters are as follows: carrier frequency 15MHz, sampling rate 60MHz, symbol rate 3M Baud, modulation mode BPSK, SNR 10dB. The signal is sampled after a band-pass filter with bandwidth of 20MHz, and then gets $r(n)$. The spectrum of $r(n)$ are shown in Figure 2. If we use SVD to estimate SNR of $r(n)$, the result is far from the actual value. Because SVD requires the noise power spectrum keeping flat in the whole frequency range while the band-pass filter changes the noise bandwidth.

As shown in Figure 3, the noise power spectrum is no longer flat after the band-pass filter, then affects the distribution of singular values. Decompose respectively the noise before and after the band-pass filter, the singular values are shown in Figure 4. Obviously, the difference of the distribution is significant.

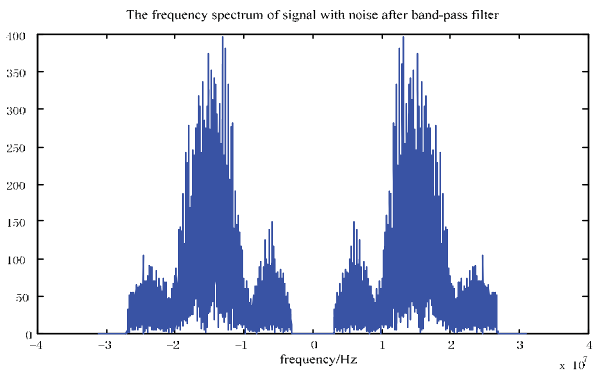

Fig.2: Spectrum of $r(n)$

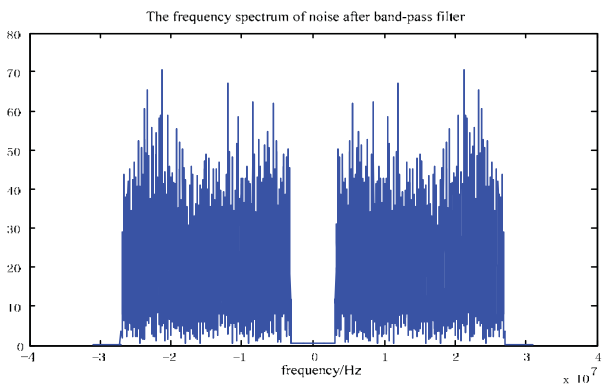

Fig.3: Spectrum of noise after band-pass filter

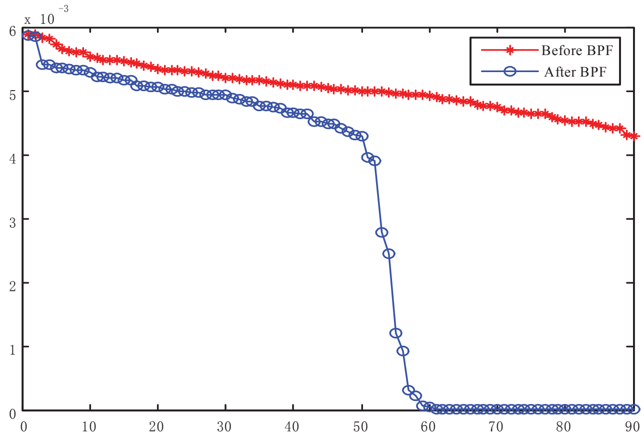

Fig.4: Singular values of noise before and after band-pass filter

Theoretically, each singular value of the gaussian white noise is σ_w^2 .As shown in Figure 4, singular values of the noise before the band-pass filter consistent with the characteristics of Gaussian white noise with uniform distribution and decreased slowly.Singular values of the noise after the band-pass filter distribute unevenly with a downward jump point. Singular values after the jump point correspond to out-of-band component of the band-pass filter.The signal dimension determined by the MDL algorithm will exceed the jump point. Finally, the estimation result will be higher than the actual value.

According to the above analysis, we must ensure the noise power spectral density being flat over the entire frequency domain before using SVD. Therefore, under the conditions of non-white noise, we must whiten the noise before estimating SNR of signal with.This artical proposes some improvements of the algorithm as follows:

Firstly, by means of Hilbert transform, to get complex baseband signal from original band-pass signal as follows

$$r_L(n) = hilbert(r(n)) \cdot e^{-j2\pi f_c n / f_s} \qquad (14)$$

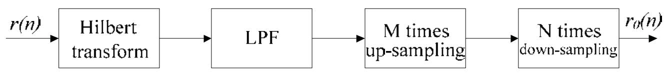

Fig. 5: Noise-whitening schematic

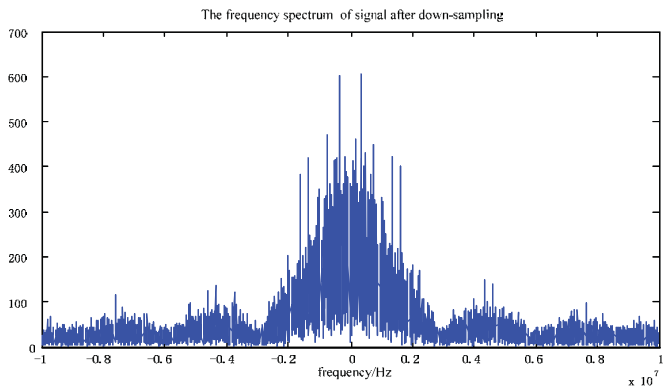

Fig.6: The spectrum of signal after noise whitening

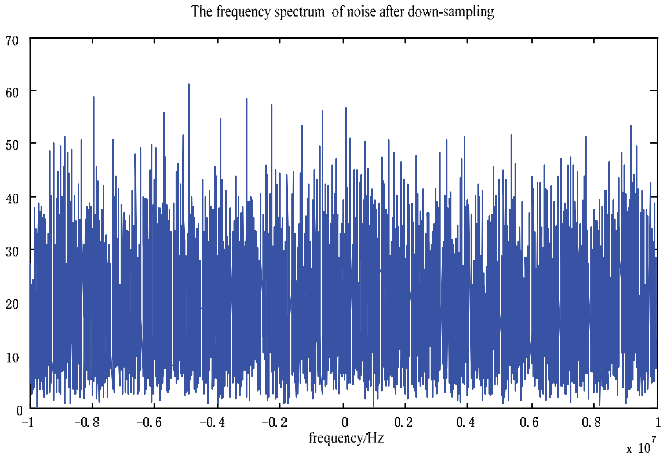

Fig.7: The spectrum of noise after whitening

Secondly, use a low-pass filter with bandwidth of W to band-limited filtered the complex baseband signal. The order of the filter should be high enough to ensure that the transition is so narrow that can be approximated as an ideal low-pass filter.The bandwidth of the filter should be lower than half of original band-pass filter and satisfies the formula $W = Mf_s/2N$, both M and N are integers. Specific values may be determined according to the actual needs or experience.

Thirdly, M times up-sampling and N times down-sampling the band-limited signal above to make the signal spectrum expansion of the entire frequency range. At the same time, the noise power spectral density becomes flat over the entire frequency domain. Then estimate the SNR by using the SVD.

All the steps are shown in Figure 5, respectively to deal with the signal and noise, spectrum of the results is shown in Figure 6 and Figure 7.We can see that spectrum of the processed signal expands to the whole frequency domain. Spectrum of the processed noise is flat enough to be considered as white gaussian noise.

4 Simulation and analysis

After processing in section 3 of this paper, we get $r_0(n)$ with white gaussian noise. Then estimate SNR of $r_0(n)$ by using SVD, compare the result with SNR estimation of $r(n)$. Simulation parameters are: M=1， N=3, SNR is 0~40dB, L=300. Use three kinds of modulation mode and do 100 independent simulations. Results are shown in Figure 8~11.

We can see that SNR estimations without noise whitening are very different from the actual. After noise whitening, the estimation error is less than 2dB when SNR is 0 ~ 30dB. It can be concluded as following: using MSVD can get accurate estimate result in larger SNR range, estimation error is small. The MSVD algorithm has certain reference significance and practical value.

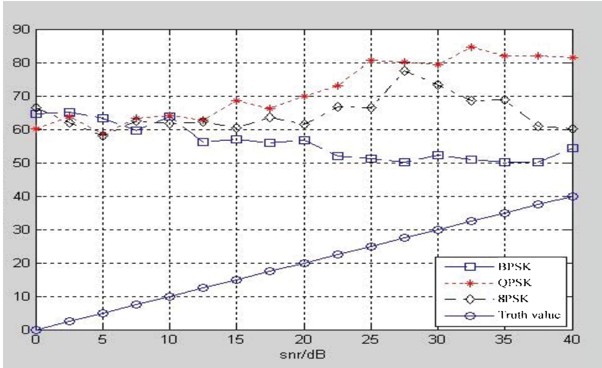

Figure 8 SNR estimation mean value curve
（without noise whitening）

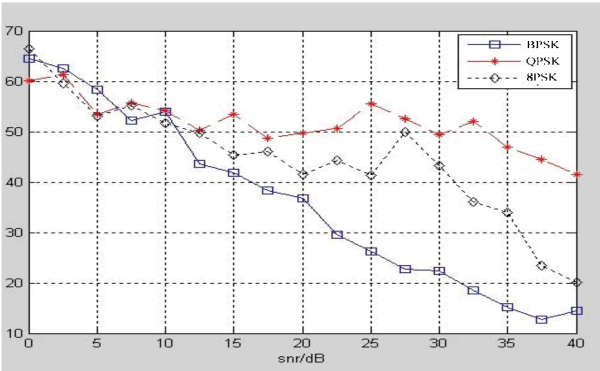

Figure 9 SNR estimation error curve
（without noise whitening）

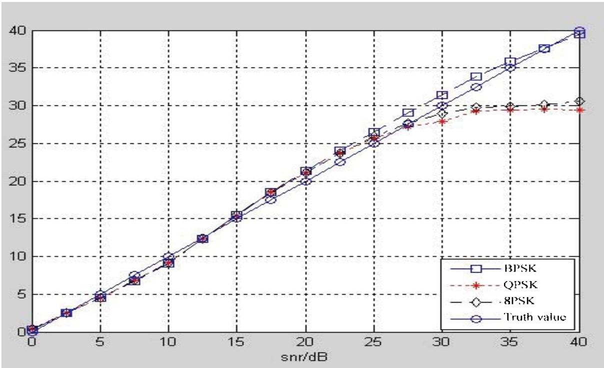

Figure 10 SNR estimation mean value curve
（with noise whitening）

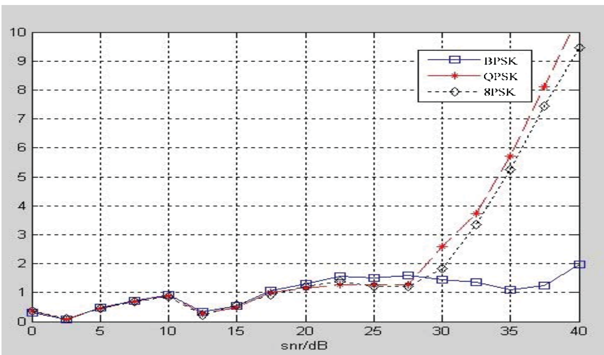

Figure 11 SNR estimation error curve
（with noise whitening）

5 Conclusion

This paper studies the method of SNR estimation in spaceflight TT&C system and proposes a modified SNR estimation algorithm based on singular value decomposition. This algorithm expands the scope by noise whitening. Simulation results show that the algorithm has great accuracy and stability when SNR is 0 ~ 30dB.

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